Rank Ordering and Parameter Contributions of Parallel Vibration Transfer Path Systems*

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Abstract
This paper based on basic theory of vibration and sensitivity technique, the problem for rank ordering and parameter contribution rank of parallel vibration transfer paths are solved. The transfer ratio and sensitivity are employed as measurement criterion, and the effective approach for rank ordering and parameter contribution rank of parallel vibration transfer paths are presented. The rank ordering of contributions among multiple and/or multi-dimensional paths are interpreted correctly and expressly in frequency range.

Key words: Vibration, Transmission, Path, Rank; Parameter contributions, Frequency and time range

1. Introduction

It is valuable to give investigations on dynamic interactions among multiple and/or multi-dimensional paths in many practical structures and machines. The effects of sources, transfer paths, receivers and dynamic interactions among them, on the overall vibration and noise performance are not well understood. Many elastic path elements exhibit multi-dimensional aspects beyond very low frequencies and thus the complexity of the problem increases. Identification of dominant path(s) in any system (or even a sub-system) is crucial for subsequent structural modifications or component selection in order to attenuate vibration and noise levels. However, a correct interpretation of the contributions of multiple paths is not straightforward, especially at low and mid frequencies. For example, conventional measures such as force and velocity transmissibilities may not be applicable to multi-dimensional problems since the associated transmissibility terms must be described using a matrix though the units of off-diagonal terms are not compatible [1]. Further, the existing methods of transfer path analysis, based on mobility or impedance formulation, seem to yield inconsistent results in terms of rank ordering and their applications are cumbersome [1]. Vibratory energy (or power) analysis methods have been widely used to describe the dynamic behavior of structural and acoustic systems [1-6]. Both deterministic and statistical analyses have been employed to cover a broad range of frequencies even though some asymptotic methods are applicable only in the higher frequency regimes [6].

On the basis of the basic conception of vibration theories and the sensitivity technique of structures, this paper, practically and effectively, presents the quantitative method for path transmissibility of parallel vibration transfer path systems in frequency range, and the parameter contribution rank of parallel vibration transfer path systems in time range with dynamic sensitivity as measurement criterion. These can be conveniently used in describing the rank order of multiple and/or multi-dimensional paths, and furthermore the theoretical analysis and numerical computation for path transmissibility and sensitivity of parallel vibration transfer path systems can be achieved, so as to provide effective approach to decease vibration and noise by means of modifying structure topology or model parameter and selecting or changing parts.

2. Transmissibility of the source

Considering the two-degree-of-freedom parallel vibration transfer path system (Fig.1), where only single excitation is considered. Governing equation of Fig. 1 is shown as follow

$$\boldsymbol{M}\ddot{\boldsymbol{x}}+\boldsymbol{C}\dot{\boldsymbol{x}}+\boldsymbol{K}\boldsymbol{x}=\boldsymbol{F}(t) \tag{1}$$

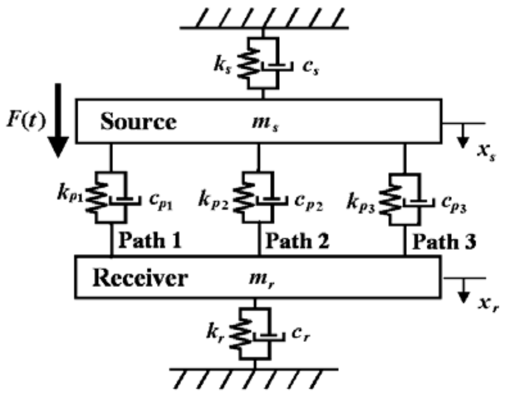

Fig. 1. The two-degree-of-freedom parallel vibration transfer path system model

where

$$\boldsymbol{M}=\begin{bmatrix} m_s & 0 \\ 0 & m_r \end{bmatrix},\quad \boldsymbol{C}=\begin{bmatrix} c_s+c_p & -c_p \\ -c_p & c_r+c_p \end{bmatrix},\quad \boldsymbol{K}=\begin{bmatrix} k_s+k_p & -k_p \\ -k_p & k_r+k_p \end{bmatrix}$$

$$\boldsymbol{x}=\begin{Bmatrix} x_s \\ x_r \end{Bmatrix},\quad \boldsymbol{F}(t)=\begin{Bmatrix} F_0\mathrm{e}^{\mathrm{i}\omega t} \\ 0 \end{Bmatrix}$$

where $c_p=c_{p1}+c_{p2}+c_{p3}$, $k_p=k_{p1}+k_{p2}+k_{p3}$. Since $\{F(t)\}$ is sinusoidal excitations

$$F_s(t)=F_0\mathrm{e}^{\mathrm{i}\omega t},\quad F_r(t)=0 \tag{2}$$

Assume responses are

$$x_s(t)=X_s\mathrm{e}^{\mathrm{i}(\omega t-\varphi)},\quad x_r(t)=X_r\mathrm{e}^{\mathrm{i}(\omega t-\varphi)} \tag{3}$$

where X_S and X_r depend on and physical parameters. Substitute (2) and (3) into (1)

$$\begin{bmatrix} -\omega^2 m_s+\mathrm{i}\omega(c_s+c_p)+(k_s+k_p) & -\mathrm{i}\omega c_p-k_p \\ -\mathrm{i}\omega c_p-k_p & -\omega^2 m_r+\mathrm{i}\omega(c_r+c_p)+(k_r+k_p) \end{bmatrix}\begin{Bmatrix} X_s\mathrm{e}^{-\mathrm{i}\varphi} \\ X_r\mathrm{e}^{-\mathrm{i}\varphi} \end{Bmatrix}=\begin{Bmatrix} F_0 \\ 0 \end{Bmatrix} \tag{4}$$

The frequency function is

$$\det[Z(\omega)]=\det\begin{bmatrix} -\omega^2 m_s+\mathrm{i}\omega(c_s+c_p)+(k_s+k_p) & -\mathrm{i}\omega c_p-k_p \\ -\mathrm{i}\omega c_p-k_p & -\omega^2 m_r+\mathrm{i}\omega(c_r+c_p)+(k_r+k_p) \end{bmatrix}$$

$$=\omega^4 m_s m_r-\omega^2\left[m_s(k_r+k_p)+(c_s+c_p)(c_r+c_p)+m_r(k_s+k_p)-c_p^2\right]+(k_s+k_p)(k_r+k_p)-k_p^2$$

$$+\mathrm{i}\left\{-\omega^3\left[m_s(c_r+c_p)+m_r(c_s+c_p)\right]+\omega\left[(c_s+c_p)(k_r+k_p)+(c_r+c_p)(k_s+k_p)-2c_pk_p\right]\right\} \tag{5}$$

where

$$\mathrm{Re}\{\det[Z(\omega)]\}$$

$$=\omega^4 m_s m_r-\omega^2\left[m_s(k_r+k_p)+(c_s+c_p)(c_r+c_p)+m_r(k_s+k_p)-c_p^2\right]+(k_s+k_p)(k_r+k_p)-k_p^2$$

$$\mathrm{Im}\{\det[Z(\omega)]\}$$

$$=-\omega^3\left[m_s(c_r+c_p)+m_r(c_s+c_p)\right]+\omega\left[(c_s+c_p)(k_r+k_p)+(c_r+c_p)(k_s+k_p)-2c_pk_p\right]$$

Therefore, the magnitudes of steady state response are

$$X_s(\omega)\mathrm{e}^{-\mathrm{i}\varphi} = \frac{\left[-\omega^2 m_r + \mathrm{i}\omega\left(c_r + c_p\right) + \left(k_r + k_p\right)\right]F_0}{\det[Z(\omega)]} \tag{6a}$$

$$X_r(\omega)\mathrm{e}^{-\mathrm{i}\varphi} = \frac{\left(\mathrm{i}\omega c_p + k_p\right)F_0}{\det[Z(\omega)]} \tag{6b}$$

Hence

$$X_s = \left|X_s(\omega)\mathrm{e}^{-\mathrm{i}\varphi}\right| = F_0\sqrt{\frac{\left[-\omega^2 m_r + \left(k_r + k_p\right)\right]^2 + \left[\omega\left(c_r + c_p\right)\right]^2}{\left(\mathrm{Re}\{\det[Z(\omega)]\}\right)^2 + \left(\mathrm{Im}\{\det[Z(\omega)]\}\right)^2}} \tag{7a}$$

$$X_r = \left|X_r(\omega)\mathrm{e}^{-\mathrm{i}\varphi}\right| = F_0\sqrt{\frac{k_p^2 + \left(\omega c_p\right)^2}{\left(\mathrm{Re}\{\det[Z(\omega)]\}\right)^2 + \left(\mathrm{Im}\{\det[Z(\omega)]\}\right)^2}} \tag{7b}$$

The interfacial forces are

$$F_{Tpj} = k_{pj}\left(x_r - x_s\right) + c_{pj}\left(\dot{x}_r - \dot{x}_s\right) = \left[k_{pj}\left(X_r - X_s\right) + \mathrm{i}\omega c_{pj}\left(X_r - X_s\right)\right]\mathrm{e}^{\mathrm{i}(\omega t - \varphi)}$$
$$= \left(k_{pj} - \mathrm{i}\omega c_{pj}\right)\left(X_r - X_s\right)\mathrm{e}^{\mathrm{i}(\omega t - \varphi)} \quad (j=1,2,3) \tag{8}$$

Their magnitudes

$$\left|F_{Tp_j}\right| = F_0\sqrt{\frac{\left(k_{pj}^2 + \omega^2 c_{pj}^2\right)\left[\omega^2 c_r^2 + \left(k_r - \omega^2 m_r\right)^2\right]}{\left(\mathrm{Re}\{\det[Z(\omega)]\}\right)^2 + \left(\mathrm{Im}\{\det[Z(\omega)]\}\right)^2}} \quad (j=1,2,3) \tag{9}$$

The transmissibilities of transmitted force and excitation forces are

$$\left|\frac{F_{Tp_j}}{F_0}\right| = \sqrt{\frac{\left(k_{pj}^2 + \omega^2 c_{pj}^2\right)\left[\omega^2 c_r^2 + \left(k_r - \omega^2 m_r\right)^2\right]}{\left(\mathrm{Re}\{\det[Z(\omega)]\}\right)^2 + \left(\mathrm{Im}\{\det[Z(\omega)]\}\right)^2}} \quad (j=1,2,3) \tag{10}$$

3. Analysis of sensitivity of the paths

Considering the five-degree-of-freedom parallel vibration transfer path system model in Fig.2, where only single excitation is considered, and the mass matrix, damping matrix, stiffness matrix, displacement vector and external force vector are respectively

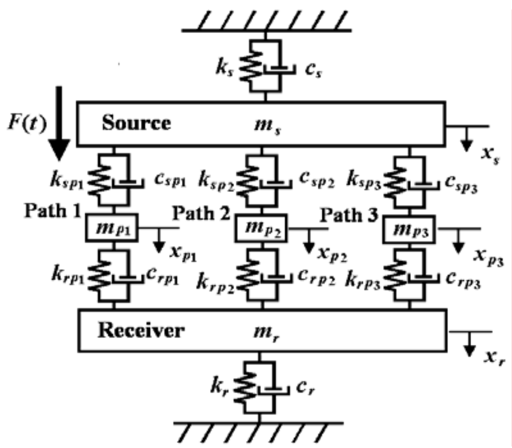

Fig. 2. The five-degree-of-freedom parallel vibration transfer path system model

$$[M] = \mathrm{diag}\left[m_s\ m_{p1}\ m_{p2}\ m_{p3}\ m_r\right]$$

$$[C] = \begin{bmatrix} c_s + c_{sp1} + c_{sp2} + c_{sp3} & -c_{sp1} & -c_{sp2} & -c_{sp3} & 0 \\ -c_{sp1} & c_{sp1} + c_{rp1} & 0 & 0 & -c_{rp1} \\ -c_{sp2} & 0 & c_{sp2} + c_{rp2} & 0 & -c_{rp2} \\ -c_{sp3} & 0 & 0 & c_{sp3} + c_{rp3} & -c_{rp3} \\ 0 & -c_{rp1} & -c_{rp2} & -c_{rp3} & c_r + c_{rp1} + c_{rp2} + c_{rp3} \end{bmatrix}$$

$$[K]=\begin{bmatrix} k_s+k_{sp1}+k_{sp2}+k_{sp3} & -k_{sp1} & -k_{sp2} & -k_{sp3} & 0 \\ -k_{sp1} & k_{sp1}+k_{rp1} & 0 & 0 & -k_{rp1} \\ -k_{sp2} & 0 & k_{sp2}+k_{rp2} & 0 & -k_{rp2} \\ -k_{sp3} & 0 & 0 & k_{sp3}+k_{rp3} & -k_{rp3} \\ 0 & -k_{rp1} & -k_{rp2} & -k_{rp3} & k_r+k_{rp1}+k_{rp2}+k_{rp3} \end{bmatrix}$$

$$\{F(t)\}=\{F_0\sin(\omega t)\,0\,0\,0\,0\}^{\mathrm{T}},\quad \{x(t)\}=\{x_s\ x_{p1}\ x_{p2}\ x_{p3}\ x_r\}^{\mathrm{T}}$$

Take differentiation of equation (1) to get the following sensitivity equation

$$\boldsymbol{M}\frac{\mathrm{D}\ddot{\boldsymbol{x}}}{\mathrm{D}\boldsymbol{V}^{\mathrm{T}}}+\boldsymbol{C}\frac{\mathrm{D}\dot{\boldsymbol{x}}}{\mathrm{D}\boldsymbol{V}^{\mathrm{T}}}+\boldsymbol{K}\frac{\mathrm{D}\boldsymbol{x}}{\mathrm{D}\boldsymbol{V}^{\mathrm{T}}}=\frac{\boldsymbol{F}(t)}{\partial\boldsymbol{V}^{\mathrm{T}}}-\frac{\partial\boldsymbol{M}}{\partial\boldsymbol{V}^{\mathrm{T}}}(\boldsymbol{I}_s\otimes\ddot{\boldsymbol{x}})-\frac{\partial\boldsymbol{C}}{\partial\boldsymbol{V}^{\mathrm{T}}}(\boldsymbol{I}_s\otimes\dot{\boldsymbol{x}})-\frac{\partial\boldsymbol{K}}{\partial\boldsymbol{V}^{\mathrm{T}}}(\boldsymbol{I}_s\otimes\boldsymbol{x}) \tag{11}$$

where $\boldsymbol{I}_S$ is the identity matrix with degree s, the symbol $\otimes$ is "Kronecker" product, $\boldsymbol{V}=(m_{p1}\ m_{p2}\ m_{p3}\ c_{sp1}\ c_{sp2}\ c_{sp3}\ c_{rp1}\ c_{rp2}\ c_{rp3}\ k_{sp1}\ k_{sp2}\ k_{sp3}\ k_{rp1}\ k_{rp2}\ k_{rp3})^{\mathrm{T}}$, $\mathrm{D}\boldsymbol{x}/\mathrm{D}\boldsymbol{V}^{\mathrm{T}}$, $\mathrm{D}\dot{\boldsymbol{x}}/\mathrm{D}\boldsymbol{V}^{\mathrm{T}}$, $\mathrm{D}\ddot{\boldsymbol{x}}/\mathrm{D}\boldsymbol{V}^{\mathrm{T}}$ is the Jacobi matrix, i.e. the sensitivity matrix. Solve for $\boldsymbol{x}$, $\dot{\boldsymbol{x}}$, $\ddot{\boldsymbol{x}}$ from Eq. (1) and substitute the solutions into equation (11), to get the sensitivity matrix $\mathrm{D}\boldsymbol{x}/\mathrm{D}\boldsymbol{V}^{\mathrm{T}}$, $\mathrm{D}\dot{\boldsymbol{x}}/\mathrm{D}\boldsymbol{V}^{\mathrm{T}}$, $\mathrm{D}\ddot{\boldsymbol{x}}/\mathrm{D}\boldsymbol{V}^{\mathrm{T}}$, so that we can analyze the path sensitivity. Based on specific situations, we can represent the sensitivity of each path as

$$\frac{\mathrm{D}\boldsymbol{x}}{\mathrm{D}m_p}=\frac{\partial\boldsymbol{x}}{\partial m_{p1}}+\frac{\partial\boldsymbol{x}}{\partial m_{p2}}+\frac{\partial\boldsymbol{x}}{\partial m_{p3}} \tag{12a}$$

$$\frac{\mathrm{D}\boldsymbol{x}}{\mathrm{D}c_{sp}}=\frac{\partial\boldsymbol{x}}{\partial c_{sp1}}+\frac{\partial\boldsymbol{x}}{\partial c_{sp2}}+\frac{\partial\boldsymbol{x}}{\partial c_{sp3}} \tag{12b}$$

$$\frac{\mathrm{D}\boldsymbol{x}}{\mathrm{D}c_{rp}}=\frac{\partial\boldsymbol{x}}{\partial c_{rp1}}+\frac{\partial\boldsymbol{x}}{\partial c_{rp2}}+\frac{\partial\boldsymbol{x}}{\partial c_{rp3}} \tag{12c}$$

$$\frac{\mathrm{D}\boldsymbol{x}}{\mathrm{D}k_{sp}}=\frac{\partial\boldsymbol{x}}{\partial k_{sp1}}+\frac{\partial\boldsymbol{x}}{\partial k_{sp2}}+\frac{\partial\boldsymbol{x}}{\partial k_{sp3}} \tag{12d}$$

$$\frac{\mathrm{D}\boldsymbol{x}}{\mathrm{D}k_{rp}}=\frac{\partial\boldsymbol{x}}{\partial k_{rp1}}+\frac{\partial\boldsymbol{x}}{\partial k_{rp2}}+\frac{\partial\boldsymbol{x}}{\partial k_{rp3}} \tag{12e}$$

We can use these to determine the contribution of the parameters of each path. If the sensitivity is higher, the response and its contribution will be higher.

4. Numerical examples

Example 1：2-DOF system as Fig. 1. ms=8 kg, cs=0.5 N·s/m, ks=100 N/m, mr=10 kg, cr=1.0 N·s/m, kr=180 N/m, cp1=0.5 N·s/m, cp2=0.75 N·s/m, cp3=0.25 N·s/m, kp1=450 N/m, kp2=225 N/m, kp3=300 N/m, F0=100 N, ω=100 rad/s. Determine the transmissibility and sensitivity of each path.

The natural frequencies and natural modes are

$$\omega_1=3.9397,\quad \{X^{(1)}\}=\begin{Bmatrix}1.0000\\0.9752\end{Bmatrix},\quad \omega_2=15.3086,\quad \{X^{(2)}\}=\begin{Bmatrix}1.0000\\-0.8203\end{Bmatrix}$$

Fig 3 shows the transmissibilities of $|F_{Tpj}/F_0|$ (j=1,2,3) of each path as functions of excitation frequency ω (rad/s).

First, at two natural frequencies ($\omega=\omega_1$ or $\omega=\omega_2$), $|F_{Tpj}/F_0|$ (j=1,2,3) are large; Second path 1 has larger contribution compared with path 3, and path 3 has larger contribution than path 2. Therefore, we can design vibration controls accordingly.

Example 2: 5-DOF system. ms=0.5, cs=1 N·s/m, ks=500 N/m, mr=0.5 kg, cr=1 N·s/m, kr=1000 N/m, mp1=0.4 kg, mp2=0.5 kg, mp3=0.6 kg, csp1=crp1=6 N·s/m, csp2=crp2=4 N·s/m, csp3=crp3=8 N·s/m, ksp1=krp1=800 N/m, ksp2=krp2=600 N/m, ksp3=krp3=400 N/m, F0=10 N, ω=10 rad/s. Determine system characteristics and responses.

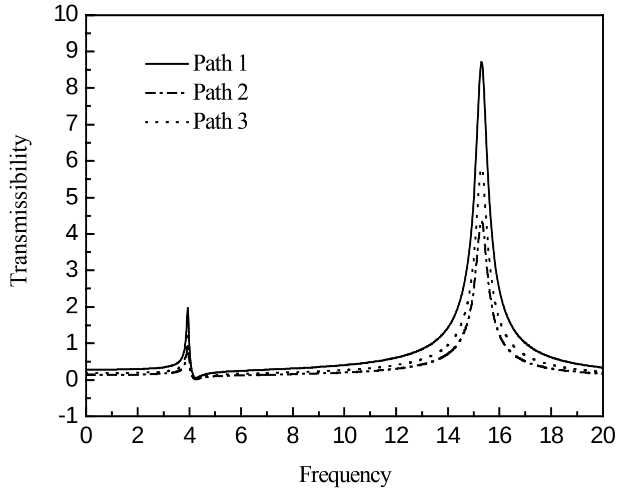

Fig. 3. The transmissibility curve

Figs. 4-6 show the sensitivities V=(mp cp kp)T as functions of time.

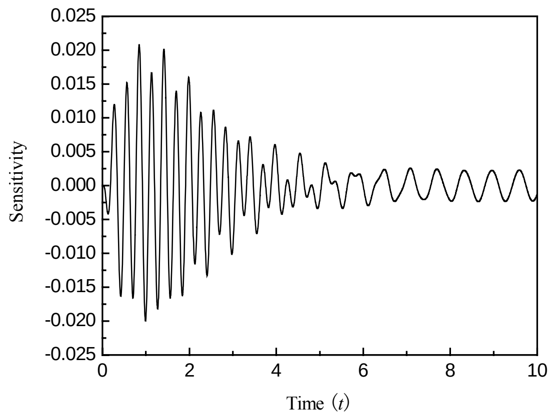

Fig.4. The curve of the sensitivity with respect to mass

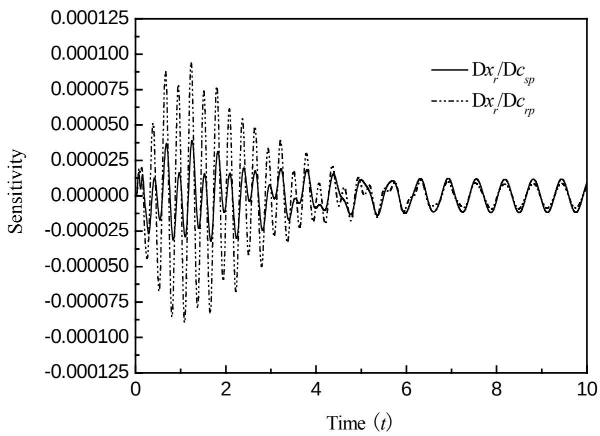

Fig.5. The curve of the sensitivity with respect to damping

We can see that mass has largest contribution to the receiver, then is the damping and stiffness is the least. Furthermore, at the beginning, the paths closer to the receivers have larger contribution compared with those which are farther away. Therefore, we need to design mass, stiffness and damping carefully.

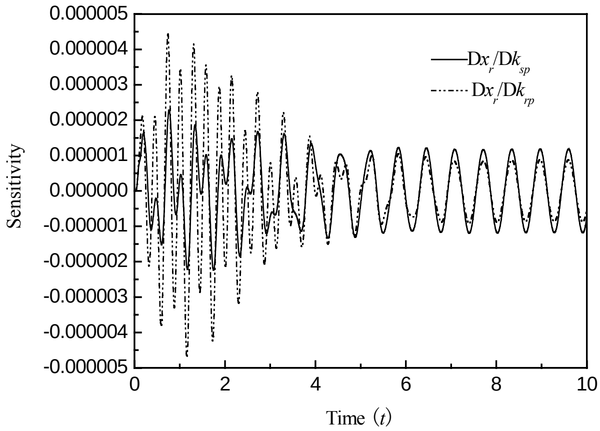

Fig.6. The curve of the sensitivity with respect to stiffness

5. Conclusion

This paper deals with the contribution and ranking of transmission path. Based on the basic linear vibration theory, transmissibility and sensitivity are using as criteria to give the rank ordering of path in the frequency domain as well as in the time domain. This provides insights into the TPA and path ranking.

6. References

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